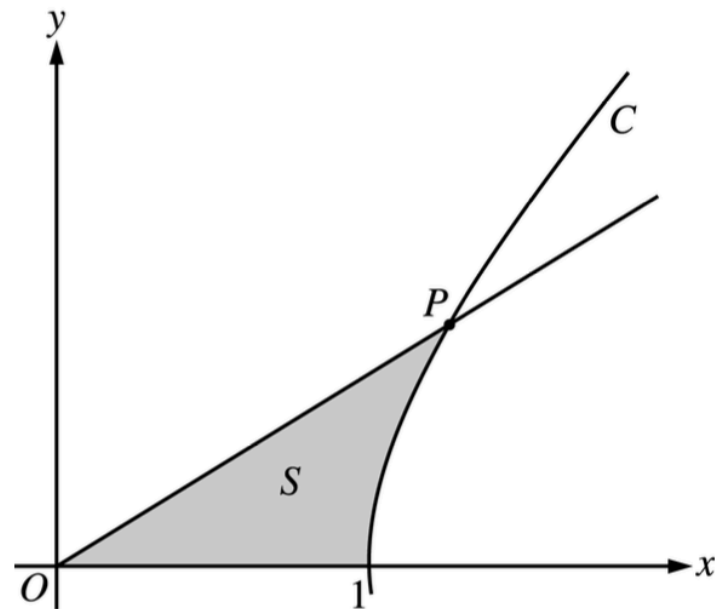
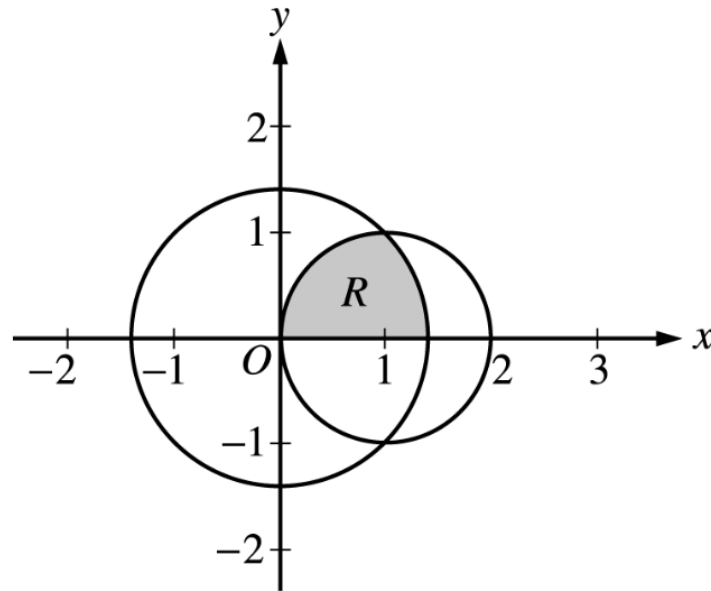


2003 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS



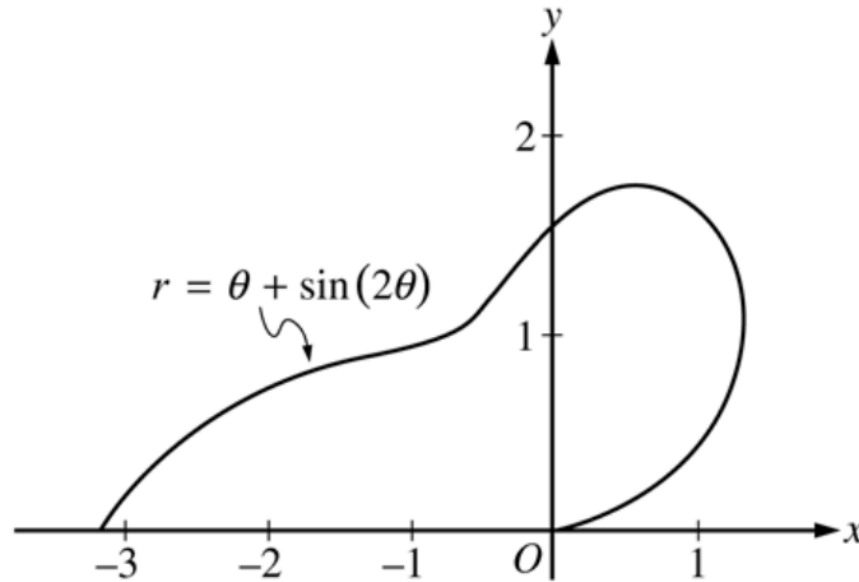
3. The figure above shows the graphs of the line $x = \frac{5}{3}y$ and the curve C given by $x = \sqrt{1 + y^2}$. Let S be the shaded region bounded by the two graphs and the x -axis. The line and the curve intersect at point P .
- (a) Find the coordinates of point P and the value of $\frac{dx}{dy}$ for curve C at point P .
- (b) Set up and evaluate an integral expression with respect to y that gives the area of S .
- (c) Curve C is a part of the curve $x^2 - y^2 = 1$. Show that $x^2 - y^2 = 1$ can be written as the polar equation
- $$r^2 = \frac{1}{\cos^2 \theta - \sin^2 \theta}.$$
- (d) Use the polar equation given in part (c) to set up an integral expression with respect to the polar angle θ that represents the area of S .

2003 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)



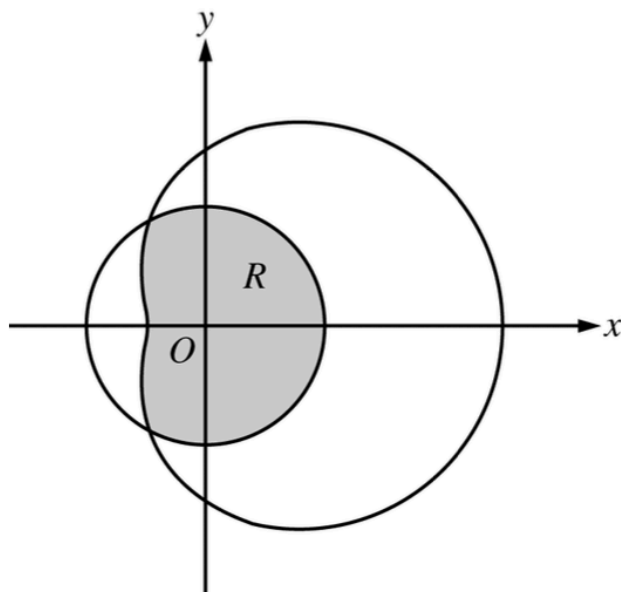
2. The figure above shows the graphs of the circles $x^2 + y^2 = 2$ and $(x - 1)^2 + y^2 = 1$. The graphs intersect at the points $(1, 1)$ and $(1, -1)$. Let R be the shaded region in the first quadrant bounded by the two circles and the x -axis.
- Set up an expression involving one or more integrals with respect to x that represents the area of R .
 - Set up an expression involving one or more integrals with respect to y that represents the area of R .
 - The polar equations of the circles are $r = \sqrt{2}$ and $r = 2 \cos \theta$, respectively. Set up an expression involving one or more integrals with respect to the polar angle θ that represents the area of R .

2005 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS



2. The curve above is drawn in the xy -plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \leq \theta \leq \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.
- (a) Find the area bounded by the curve and the x -axis.
- (b) Find the angle θ that corresponds to the point on the curve with x -coordinate -2 .
- (c) For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r ? What does this fact say about the curve?
- (d) Find the value of θ in the interval $0 \leq \theta \leq \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

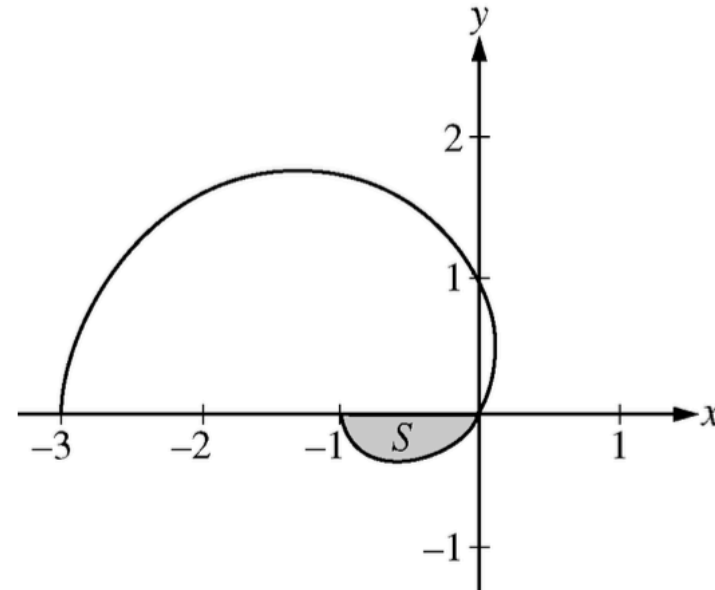
2007 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS



3. The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.
- (a) Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2\cos\theta$, as shaded in the figure above. Find the area of R .
- (b) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos\theta$ has position $(x(t), y(t))$ at time t , with $\theta = 0$ when $t = 0$. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.
- (c) For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

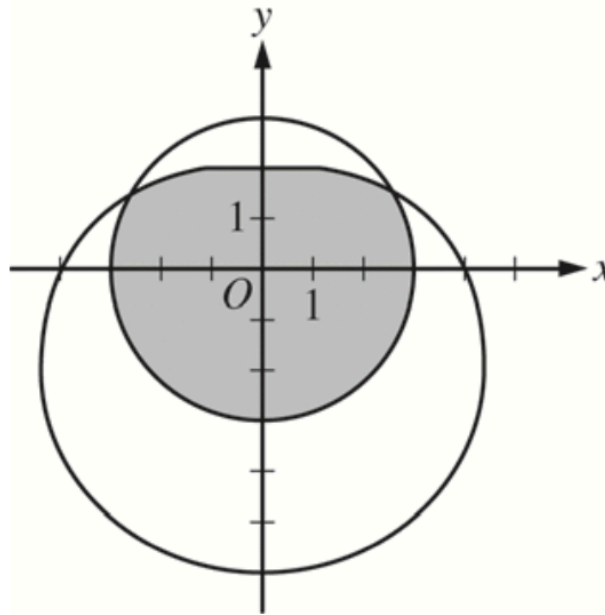
No calculator is allowed for these problems.



4. The graph of the polar curve $r = 1 - 2\cos\theta$ for $0 \leq \theta \leq \pi$ is shown above. Let S be the shaded region in the third quadrant bounded by the curve and the x -axis.
- (a) Write an integral expression for the area of S .
- (b) Write expressions for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of θ .
- (c) Write an equation in terms of x and y for the line tangent to the graph of the polar curve at the point where $\theta = \frac{\pi}{2}$. Show the computations that lead to your answer.

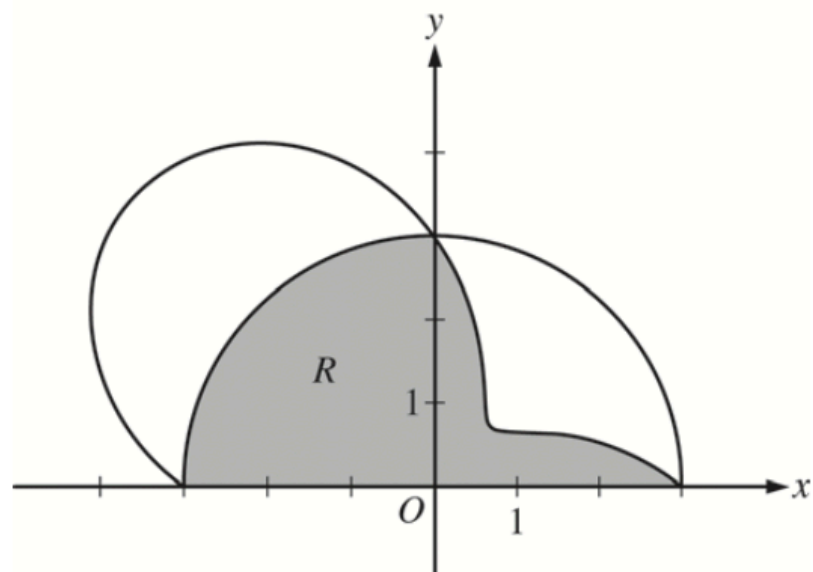
2. The polar curve r is given by $r(\theta) = 3\theta + \sin \theta$, where $0 \leq \theta \leq 2\pi$.
- Find the area in the second quadrant enclosed by the coordinate axes and the graph of r .
 - For $\frac{\pi}{2} \leq \theta \leq \pi$, there is one point P on the polar curve r with x -coordinate -3 . Find the angle θ that corresponds to point P . Find the y -coordinate of point P . Show the work that leads to your answers.
 - A particle is traveling along the polar curve r so that its position at time t is $(x(t), y(t))$ and such that $\frac{d\theta}{dt} = 2$. Find $\frac{dy}{dt}$ at the instant that $\theta = \frac{2\pi}{3}$, and interpret the meaning of your answer in the context of the problem.

2013 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS



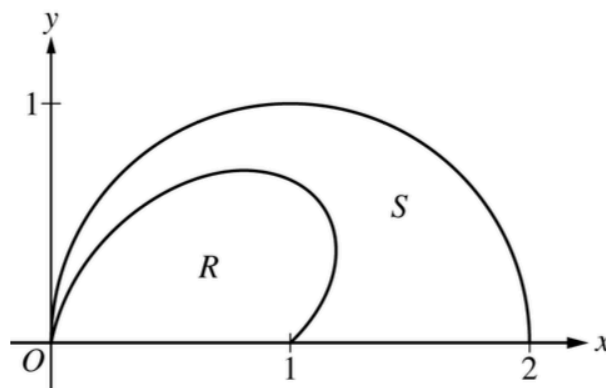
2. The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.
- (a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S .
- (b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \leq t \leq 2$ for which the x -coordinate of the particle's position is -1 .
- (c) For the particle described in part (b), find the position vector in terms of t . Find the velocity vector at time $t = 1.5$.

2014 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS



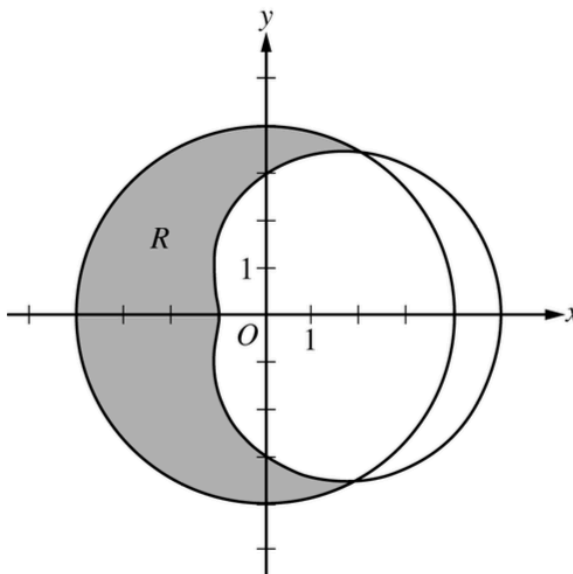
2. The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \leq \theta \leq \pi$.
- (a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R .
- (b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.
- (c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.
- (d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \geq 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.

2017 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS



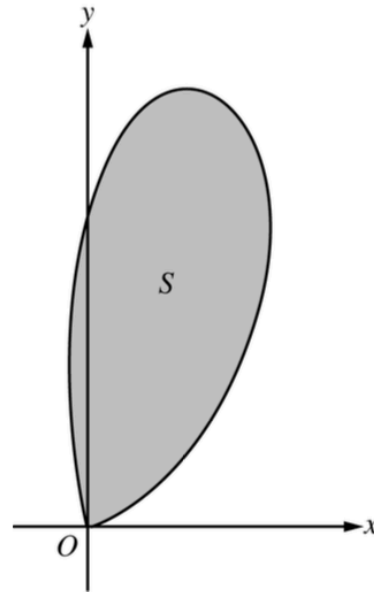
2. The figure above shows the polar curves $r = f(\theta) = 1 + \sin \theta \cos(2\theta)$ and $r = g(\theta) = 2 \cos \theta$ for $0 \leq \theta \leq \frac{\pi}{2}$. Let R be the region in the first quadrant bounded by the curve $r = f(\theta)$ and the x -axis. Let S be the region in the first quadrant bounded by the curve $r = f(\theta)$, the curve $r = g(\theta)$, and the x -axis.
- (a) Find the area of R .
- (b) The ray $\theta = k$, where $0 < k < \frac{\pi}{2}$, divides S into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k .
- (c) For each θ , $0 \leq \theta \leq \frac{\pi}{2}$, let $w(\theta)$ be the distance between the points with polar coordinates $(f(\theta), \theta)$ and $(g(\theta), \theta)$. Write an expression for $w(\theta)$. Find w_A , the average value of $w(\theta)$ over the interval $0 \leq \theta \leq \frac{\pi}{2}$.
- (d) Using the information from part (c), find the value of θ for which $w(\theta) = w_A$. Is the function $w(\theta)$ increasing or decreasing at that value of θ ? Give a reason for your answer.

2018 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS



5. The graphs of the polar curves $r = 4$ and $r = 3 + 2 \cos \theta$ are shown in the figure above. The curves intersect at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.
- (a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2 \cos \theta$, as shown in the figure above. Write an expression involving an integral for the area of R .
- (b) Find the slope of the line tangent to the graph of $r = 3 + 2 \cos \theta$ at $\theta = \frac{\pi}{2}$.
- (c) A particle moves along the portion of the curve $r = 3 + 2 \cos \theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.

2019 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS



2. Let S be the region bounded by the graph of the polar curve $r(\theta) = 3\sqrt{\theta} \sin(\theta^2)$ for $0 \leq \theta \leq \sqrt{\pi}$, as shown in the figure above.
- (a) Find the area of S .
- (b) What is the average distance from the origin to a point on the polar curve $r(\theta) = 3\sqrt{\theta} \sin(\theta^2)$ for $0 \leq \theta \leq \sqrt{\pi}$?
- (c) There is a line through the origin with positive slope m that divides the region S into two regions with equal areas. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of m .
- (d) For $k > 0$, let $A(k)$ be the area of the portion of region S that is also inside the circle $r = k \cos \theta$. Find $\lim_{k \rightarrow \infty} A(k)$.